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CHARGED STRINGY BLACK HOLES WITH NON–ABELIAN HAIR

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Abstract

Static spherically symmetric asymptotically flat charged black hole solutions are constructed within the magnetic $SU(3)$ sector of the 4-dimensional heterotic string effective action. They possess non–abelian hair in addition to the Coulomb magnetic field and are qualitatively similar to the Einstein–Yang–Mills colored $SU(3)$ black holes except for the extremal case. In the extremality limit the horizon shrinks and the resulting geometry around the origin coincides with that of an extremal abelian dilatonic black hole with magnetic charge. Non-abelian hair exibits then typical sphaleron structure.

Stringy four-dimensional charged black holes attracted much attention recently in connection with the problem of the final stage of Hawking evaporation [1], [2], [3] . A novel feature of the $U(1)$ charged dilatonic black holes is the geometric structure of an infinite throat in the extremal limit. In the string frame the geometry is regular and this suggests some new (though still qualitative) resolution of the information puzzle [4]. This family of solutions is quite different from the supersymmetric $U^2(1)$ ones [5] possessing both electric and magnetic charges and having the standard Reissner-Nordstrom features in the extremal limit.

These results were obtained within the abelian Einstein–Maxwell–dilaton model. Within the context of some larger non–abelian gauge group, which is associated with the string theory, similar solutions can be found as embedded ones. In view of the above discussion any additional structures which can emerge in the non–abelian case are of interest. In the absense of gauge charges the existence of a two-parametric family of essentially non-abelian dilatonic black holes was shown recently [6]. From the other hand, in the pure Einstein–Yang–Mills model (without dilaton) with the $SU(3)$ group there exist charged black hole solutions possessing non-abelian hair [7]. They have non–trivial extremal limit in which non–abelian hair survives. The $SU(3)$ group is the minimal one for which non–linear superposition of a Coulomb field and a non-abelian hair is not forbidden by the non-abelian baldness theorem [8].

It can anticipated that analogous non-abelian charged black holes can exist in the string theory with appropriate modifications due to the dilaton. We show here that the bosonic effective action in the heterotic string theory admits indeed static spherically–symmetric magnetically charged black holes possessing non-abelian hair. They have, however, a different extremal limit as compared with the no-dilaton case. Instead, this limit looks quite similar to that of the abelian magnetically charged dilatonic black hole. When dilaton is added, the radius of the horizon of an abelian extremal solution tends to zero. In this limit the dilaton field and the abelian component of magnetic field both grow infinitely, while the non-abelian part remains finite.

We start with the field model which is the bosonic part of the 4-dimensional heterotic string effective action in the Einstein frame without axion field (for pure magnetic configurations the axion field is trivial, see [6]) and without Gauss-Bonnet term

$$S = \frac{1}{16\pi} \int \sqrt{-g}\,[\,m_{Pl}^2\,(-R + 2\partial_\mu\Phi\partial^\mu\Phi) - \exp(-2\Phi)\,F_{a\mu\nu}\,F_a^{\mu\nu}]\,d^4x\,, \tag{1}$$

where Φ is the dilaton, $F_{a\mu\nu}$ is the Yang-Mills curvature corresponding to the $SU(3)$ gauge group.

Consider static spherically symmetric space-time with the line element

$$ds^2 = \frac{\Delta\sigma^2}{r^2}dt^2 - \frac{r^2}{\Delta}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \tag{2}$$

where Δ and σ are functions of r.

Spherically–symmetric $SU(3)$ Yang–Mills connections may be classified in terms of embeddings of the $SU(2)$ subgroup. For our purposes the isospin-1 embedding is relevant (for more details see [7]) which in the gauge $A_0 = 0$ reads

$$gA_i = T_a\epsilon_{aij}\frac{x^j}{r^2}(1-K)\,+\,(\epsilon_{is\alpha}x_\beta\,+\,\epsilon_{is\beta}x_\alpha)\frac{x^s}{r^3}K_1\;. \tag{3}$$

Here $T_a = \frac{1}{2}(\lambda_7, -\lambda_5, \lambda_2)$ are normalized hermitean generators of the $SO(3)$ subgroup (with the standard Gell-Mann matrices λ_a), $\alpha, \beta = 1,2,3$ are matrix indices, g is the coupling constant, and K, K_1 are real–valued functions of the radial variable. In this notation x^j etc. are Cartesian coordinates related to spherical ones in a standard flat–space way. In what follows we also make an additional assumption for radial functions $K = K_1 = f/2\sqrt{2}$ which produces particular configurations with fixed value of a magnetic charge [7].

After dimensional reduction the action (1) will give

$$S = \frac{1}{2}\int dtdr\{m_Pl^2[\sigma'(\Delta/r - r) - \Phi'^2\Delta\sigma] - (2g)^{-2}\sigma F\exp(-2\Phi)\}\;, \tag{4}$$

where

$$F = 2\Delta f'^2 + f^4 - 2f^2 + 4 \tag{5}$$

and primes denote the derivatives with respect to r.

The abelian (embedded) solution corresponds to $f \equiv 1$ and its geometry reproduces the metric of Gibbons' magnetically charged dilatonic black hole [9]. The particular value of the magnetic charge encountered here (as well as in the no-dilaton version of the $SU(3)$ Einstein-Yang-Mills black holes) was pointed out by Marciano and Pagels long ago [10]. It is worse to be remarked also that structure allowing for the gauge–invariant definition of the magnetic charge (without Higgs scalars) emerges typically in the $SU(3)$ case [10] (for generalization to dyons see [11]. In the pure Yang–Mills context corresponding solutions are point-like monopoles (dyons) and they become black holes when gravity is added. From the other hand, the $SU(3)$ group is the one allowing for violation of the non-abelian baldness theorem [8]. This opens the possibility to consider non-abelian black holes possesing color charges. It should be remarked the well-known $SU(2)$ non–abelian black holes have zero color charge [12]; in this connection the name "colored" sometimes used for $SU(2)$ black holes with non-abelian hair [13] is somewhat misleading.

Variation of the action (4) gives the following set of equations for f, Φ, σ and Δ:

$$(\frac{f'\Delta\sigma\exp(-2\Phi)}{r^2})' = \frac{\sigma f(f^2-1)\exp(-2\Phi)}{r^2}\ , \tag{6}$$

$$(\Phi'\Delta\sigma)' = -\frac{R_g^2\sigma F\exp(-2\Phi)}{4r^2}\ , \tag{7}$$

$$(\ln\sigma)' = \frac{R_g^2 f'^2\exp(-2\Phi)}{2r} + r\Phi'^2\ , \tag{8}$$

$$-(\frac{\Delta}{r})' + 1 = \frac{R_g^2 F\exp(-2\Phi)}{4r^2} + \Delta\Phi'^2\ . \tag{9}$$

We will be interested in the asymptotically flat configurations specified by the asymptotic conditions $\sigma(\infty) = 1$, $\Delta/r^2 = 1 - 2M/r + (P^2 + D^2)/r^2 + O(1/r^3)$, where $O(1/r^2)$ terms are proportional to the sum of the magnetic and the dilaton charges squared (in what follows we use this geometric definition of the magnetic charge). Obviously, the Eq. 8 can be integrated to give

$$\sigma = \exp[-\int_r^\infty (\frac{R_g^2 f'^2\exp(-2\Phi)}{2r} + r\Phi'^2)dr]\,, \tag{10}$$

that can be used to reduce the system. We also assume the existence on an event horizon, $r = r_H$ being the largest root of $\Delta(r_H) = 0$. The ADM mass M of the solution may then be expressed through the Eq.9 as

$$M = M_H + \frac{1}{2}\int_{r_H}^{\infty}(\Delta\Phi'^2 + \frac{R_g^2 F\exp(-2\Phi)}{4r^2})dr\,, \tag{11}$$

where M_H is a "bare" mass of a black hole.

For further simplification we fix the scale by imposing on the dilaton field an asymptotic condition $\Phi(\infty) = 0$. Then using Eq.7 one can express the dilaton charge of the configuration as follows

$$D = \lim_{r\to\infty}(-r^2\Phi'(r)) = R_g^2\int_{r_H}^{\infty}\frac{\sigma F\exp(-2\Phi)}{4r^2}dr \tag{12}$$

An asymptotic behavior of the Yang-Mills function f compatible with the asymptotic flatness and corresponding to the magnetic charge $P = R_g\sqrt{3}/2$ is $f = \pm 1$. Then from the Eq. 6 it can be easily shown that everywhere outside the horizon this function is bounded $f^2 \leq 1$ [8]. This property is used in the shooting strategy to obtain the solution numerically. To implement this we first eliminate σ through the Eq. 10 from the system (6)-(9) and then solve the remaining equations in terms of power series in the vicinity of the horizon. These expansions can be parameterized by two quantities Φ_H and f_H corresponding to (finite) values of the Yang-Mills function and the dilaton field on the horizon

$$\begin{aligned} f &= f_H + \frac{f_H(f_H^2-1)}{2G_H x_H}\,y + O(y^2)\,, && (13)\\ \Phi &= \Phi_H - \frac{y}{8x_H^2}(1-\frac{1}{G_H}) + O(y^2)\,, && (14)\\ \Delta &= \frac{y}{2}\,R_g^2 G_H\,(1+O(y))\,, && (15) \end{aligned}$$

where $y = x - x_H$, $x = r^2/R_g^2$, and G_H is the horizon value of the function G (in what follows we will use R_g as the unit of length for all parameters of the corresponding dimension such as M, D, and P)

$$G = 1 - \frac{[(1-f^2)^2+3]\exp(-2\Phi)}{4x}\,. \tag{16}$$

Eliminating from the Eqs.(6)-(9) the σ-variable through the Eq. 10 one gets in terms of dimensionless variable x the following set of coupled equations for three dimensionless quantities

f, Φ and $d = \Delta/R_g^2$

$$d(f_{xx} - 2f_x\Phi_x) + \frac{1}{2}Gf_x + \frac{f(1-f^2)}{4x} = 0\ , \tag{17}$$

$$d(\Phi_{xx} + \frac{1}{x}\Phi_x) + \frac{1}{2}G\Phi_x + \frac{F\exp(-2\Phi)}{16x^2} = 0\ , \tag{18}$$

$$d_x + d(2x\Phi_x^2 - \frac{1}{2x}) + \frac{1}{2}(\frac{F\exp(-2\Phi)}{x} - 1) = 0\ . \tag{19}$$

The system consists of two equations of the second order and one of the first order, hence the solution will be fixed completely by the boundary conditions at the horizon for f, f', Φ, Φ' and d, which are parameterized according to Eqs. (13)-(15) in terms of f_H and Φ_H. The solution, like in the $SU(2)$ case [6], exists for discrete values of the parameters f_H and Φ_H, labeled by the number of zeros n of the Yang-Mills function f. For each integer n and any (non-zero) real x_H (dimensionless radius of the horizon) there exist a pair of values f_H and Φ_0, and hence we obtain two-parametric family of solutions. These values found numerically for some lower n are shown on the Table 1 for $x_H = 1$ together with the corresponding values of the σ_H, the total mass, the dilaton charge (as given by Eqs. (11), (12)) and the Hawking temperature measured in the units of R_g^{-1}

$$T = \frac{\sigma(r_H)\,G_H}{4\pi x_H}\ . \tag{20}$$

With increasing n all these quantities are likely to tend to some limiting values.

Table 1, x_H =1

n	f_H	Φ_H	σ_H	M	D	T
0	-1	0.458145	0.903508	0.790562	0.474341	0.050329
1	-0.613879	0.522050	0.847105	0.858644	0.561263	0.047310
2	-0.132473	0.548110	0.864465	0.865777	0.576706	0.046006
3	-0.021835	0.549274	0.865984	0.866018	0.577332	0.045945
4	-0.003561	0.549306	0.866025	0.866024	0.577348	0.045944

Here $n = 0$ corresponds to an (embedded) abelian $f \equiv -1$ solution. It can be seen that the value of the dilaton field on the horizon increases with growing n, while the absolute value of the

YM function f is decreasing. The dilaton charge is substantially smaller than the Scwarzschild mass.

When the radius of the horizon decreases, the horizon values of the dilaton field grow up and tend to the corresponding abelian value. At the same time the horizon values of the YM function approach the abelian value -1, and one can anticipate that "small" black holes in the vicinity of the horizon look like their abelian counterparts. At larger distances the behavior of the Yang–Mills function is qualitatively the same as previously (i.e. oscillations around zero), and the mass of the solution increases with the number of nodes. The dilaton charge becomes very close to the mass as in the case of regular stringy sphalerons [6]. The Table 2 shows the parameters of solutions up to $n = 3$ for $x_H = 0.0001$

Table 2, $x_H = 0.0001$

n	f_H	Φ_H	σ_H	M	D
0	-1	4.807898	0.016328	0.612393	0.612352
1	-0.999947	4.807956	0.016199	0.691746	0.695353
2	-0.999575	4.808081	0.016312	0.704072	0.703884
3	-0.997279	4.808844	0.016283	0.706391	0.706332

Numerical solutions for f, Φ, and σ are shown at the Figs.1–4 (all functions at the Figs.1–4 depend on variable r; $r = R_g\sqrt{x}$ and we put $R_g = 1$). The functions $\Phi(r)$ and $\sigma(r)$ are monotonic and rather similar to those in the $SU(2)$ case [6].

In the case of EYM (with no dilaton) $SU(3)$ black holes [7] there is a critical (minimal) value of x_H for which the coefficient in front of the linear in x term in the expansion of the metric function Δ near the horizon becomes zero, and consequently the leading term becomes quadratic in x. This limiting value corresponds to the extremal charged black hole with non-abelian hair and it marks a treshold horizon radius below which the family of solutions ceases to exist. In the present case situation is rather different. As we have noticed, with decreasing x_H the dilaton value on the horizon is rapidly growing up, and consequently the second (negative) term in the expression (15) is decreased with respect to the no-dilaton case. As a result, the family of solution exists now for any arbitrarily small value of the radius of the horizon.

The limiting form of solutions is of particular interest because of mentioned above intriguing properties of the corresponding abelian solution. The $U(1)$ magnetically charged extremal black hole has $D = M$, $|f| \equiv 1$ and possess the following expansions near the origin

$$\Delta = \frac{r^2}{4}(1 + \frac{r^2}{2M^2}) + O(r^6), \tag{21}$$

$$\sigma = \frac{r}{M}(1 - \frac{r^2}{2M^2}) + O(r^5), \tag{22}$$

$$\exp(-2\Phi) = \frac{2r^2}{3}(1 - \frac{r^2}{2M^2}) + O(r^6). \tag{23}$$

In order to investigate the existence of larger family of extremal magnetically charged black holes in the non-abelian case we look for somewhat more general expansions of the type (21)-(23) near the origin. From the initial system of equations (6)-(9) one can find the following family of approximate solutions

$$\Delta = \frac{r^2}{4}(1 - kr^2) + O(r^6), \tag{24}$$

$$\sigma = \sigma_0\, r(1 + kr^2) + O(r^5), \tag{25}$$

$$\exp(-2\Phi) = \frac{2r^2}{3}(1 + kr^2) + O(r^6), \tag{26}$$

$$f = -1 + br^2 + \frac{k - 3b}{4} br^4 + O(r^6), \tag{27}$$

where k, σ_0 and b are some constant real parameters. Now we use the same numerical strategy as above in order to match this expansions to the required asymptotic form of the solutions. The matching procedure fixes the discrete values of these parameters for each integer n, the number of nodes of the Yang–Mills function f . The Table 3 shows the results for $n = 1, 2, 3$, the abelian case $n = 0$ being given for comparison.

Table 3

n	k	b	σ_0	M	D
0	-1.33333	0	1.632993	0.612372	0.612372
1	-2.15396	1.076983	1.632998	0.687792	0.688132
2	-7.1271	8.502646	1.633002	0.703559	0.703916
3	-37.7495	54.43737	1.633014	0.706192	0.706561

The abelian $n = 0$ values of parameters correspond to the standard $U(1)$ extremal dilatonic black hole with the magnetic charge value $P = \sqrt{3}/2$ as precribed by our embedding into $SU(3)$ in the adopted units. The masses and dilaton charges are increased due to non–abelian hair. The YM function oscillates as depicted on the Fig. 5 interpolating between the value -1 at the horizon and $(-1)^{n+1}$ at infinity. The behavior of the metric function σ is not changed substantially as well as the dilaton field for the number of nodes considered with respect to the abelian case, see Figs. 6, 7.

In the string frame the corresponding geometry is that of an infinite throat as can be seen from the expansion

$$ds^2_{string} = \frac{3\sigma_0^2}{8} dt^2 - \frac{6}{r^2} dr^2 - \frac{3}{2}(d\theta^2 + \sin^2\theta d\phi^2). \tag{28}$$

This is the same kind of expressions as hold for the $U(1)$ extremal dilatonic black holes which are just a particular case of (28) with $\sigma_0 = 1/M$.

We conclude with the following remarks. Non–abelian embedding of the $U(1)$ magnetic stringy black holes open the possibility of additional hair structure similarly to the no-dilaton case. The role of dilaton, however, is essential in the extremality limit, in which the radius of the horizon tends to zero as in the abelian case. A novel feature of the non-abelian extremal magnetically charged dilatonic black holes is that the function f now can interpolate between the values -1 and 1 (for odd n) corresponding to topologically distinct Yang-Mills vacua. This property is quite similar to that of regular sphaleron solutions in both the EYM [14] and the EYM-dilaton [6] theory.

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Figure Captions

Fig. 1. Yang–Mills field function f for “small” black hole ($r_H = 0.01$).

Fig. 2. Yang–Mills field function f for “medium size” black hole ($r_H = 1$).

Fig. 3. Dilaton field for $r_H = 1, 0.01$ and $n = 2$.

Fig. 4. Metric function σ for $r_H = 1, 0.01$ and $n = 1, 3$.

Fig. 5. Yang–Mills function for the extremal solution, $n = 1, 2, 3$.

Fig. 6. Metric function σ for extremal abelian $n = 0$ and non–abelian $n = 3$ solutions ($n = 1, 2$ curves being between the showed ones).

Fig. 7. Dilaton field for extremal abelian and non–abelian black holes.